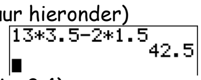
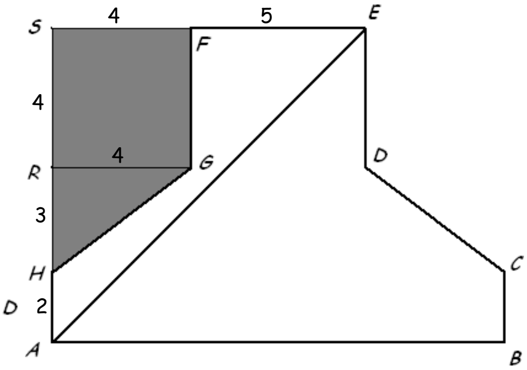
1 De oppervlakte van figuur 2.1 is de oppervlakte van een rechthoek van 7 bij 3 $\Rightarrow O = 7 \cdot 3 = 21$.
(de halve cirkel aan de bovenkant past precies in de inham aan de onderkant)

2a $O(ABCQPH) = O(ABCQPH) - O(CMQ)$ (zie de figuur hieronder)
$= 13 \cdot 3\frac{1}{2} - 2 \cdot \frac{1}{2} \cdot 2 \cdot 1\frac{1}{2} = 42\frac{1}{2}$.



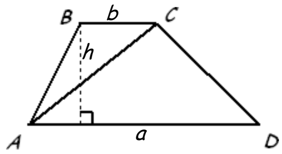
Dit is niet de helft van 73 (de oppervlakte van fig. 2.4).

2b $O(AEFGH) = O(AES) - O(RGFS) - O(HGR)$ (zie de figuur hiernaast)
$= \frac{1}{2} \cdot 9 \cdot 9 - \frac{1}{2} \cdot 3 \cdot 4 - 4 \cdot 4 = 18\frac{1}{2}$.
Dus $O(ABCDE) = 73 - 18\frac{1}{2} = 54\frac{1}{2}$.
$O(AEFGH) : O(ABCDE) = 18\frac{1}{2} : 54\frac{1}{2} = 37 : 109$.



3a $O(ABCD) = \frac{1}{2} \cdot O(AEFD) = \frac{1}{2} \cdot (a+b) \cdot h$.

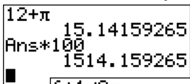
3b $O(ABCD) = O(ABC) + O(ADC)$ (zie hiernaast)
$= \frac{1}{2}ah + \frac{1}{2}bh = \frac{1}{2}h(a+b) = \frac{1}{2}(a+b)h$.



4a $O = \frac{1}{2} \cdot 4 \cdot 2 = 4$ cm^2. (neem de linkerkant als basis)

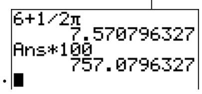
4b $O = 3 \cdot 4 - \frac{1}{2} \cdot 3 \cdot 1 - \frac{1}{2} \cdot 3 \cdot 2 - \frac{1}{2} \cdot 1 \cdot 4 = 12 - 1\frac{1}{2} - 3 - 2 = 12 - 6\frac{1}{2} = 5\frac{1}{2}$ cm^2. (teken over roosterlijnen een rechthoek er omheen)

4c $O = O_{\text{parallellogram}} + O_{\text{cirkel}} = 4 \cdot 3 + \pi \cdot 1^2 = 12 + \pi \approx 15{,}14$ cm^2 $= 1514$ mm^2.

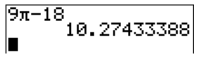


4d $O = O_{\text{trapezium}} = \frac{1}{2} \cdot (5+1) \cdot 3 = \frac{1}{2} \cdot 6 \cdot 3 = 3 \cdot 3 = 9$ cm^2.

4e $O = O_{\text{rechthoek}} + O_{\text{halve cirkel}} = 3 \cdot 2 + \frac{1}{2} \cdot \pi \cdot 1^2 = 6 + \frac{1}{2}\pi \approx 7{,}57$ cm^2 $= 757$ mm^2.



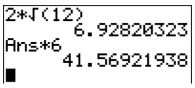
5 $O = O_{\text{cirkel}} - O_{\text{vierkant}} = O_{\text{cirkel}} - 2 \cdot O(ABC) = \pi \cdot 3^2 - 2 \cdot \frac{1}{2} \cdot 6 \cdot 3 = 9\pi - 18 \approx 10{,}27$.

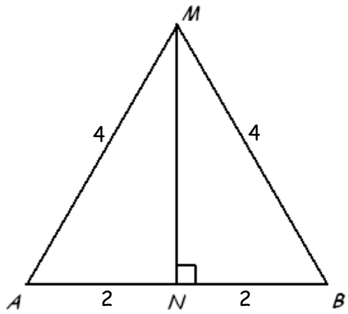


6a ΔABM: $\angle M = \angle AMB = \frac{360°}{6} = 60°$.

6b $AM = BM$ en $\angle AMB = 60° \Rightarrow \Delta ABM$ is gelijkzijdig $\Rightarrow AM = BM = AB = 4$.
ΔANM: $AN^2 + MN^2 = AM^2$
$2^2 + MN^2 = 4^2$
$MN^2 = 16 - 4 = 12 \Rightarrow MN = \sqrt{12}$.
$O(ABM) = \frac{1}{2} \cdot 4 \cdot \sqrt{12} = 2\sqrt{12} \approx 6{,}93$.

6c $O(ABCDEF) = 6 \cdot O(ABM) = 6 \cdot 2\sqrt{12} = 12\sqrt{12} \approx 41{,}6$.



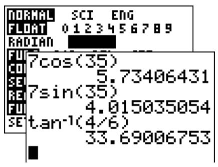


✻✻✻ **Neem GR - practicum 4 door.** (uitwerkingen aan het eind)

TOETS VOORKENNIS

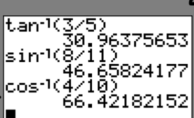
a $\cos(35°) = \frac{AB}{7} \Rightarrow AB = 7 \cdot \cos(35°) \approx 5{,}7$.
$\sin(35°) = \frac{BC}{7} \Rightarrow BC = 7 \cdot \sin(35°) \approx 4{,}0$.

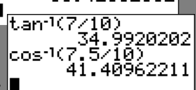
b $\tan\angle Q = \frac{4}{6} \Rightarrow \angle Q = \tan^{-1}\left(\frac{4}{6}\right) \approx 34°$.



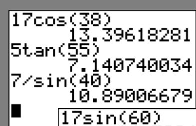
Voorkennis 1 Goniometrische verhoudingen (bladzijden 141, 142 en 143)

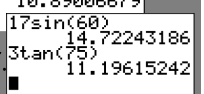
1 $\tan\angle A = \frac{3}{5} \Rightarrow \angle A = \tan^{-1}\left(\frac{3}{5}\right) \approx 31°$.
$\sin\angle D = \frac{8}{11} \Rightarrow \angle D = \sin^{-1}\left(\frac{8}{11}\right) \approx 47°$.
$\cos\angle I = \frac{4}{10} \Rightarrow \angle I = \cos^{-1}\left(\frac{4}{10}\right) \approx 66°$.
$\tan\angle K = \frac{7}{10} \Rightarrow \angle K = \tan^{-1}\left(\frac{7}{10}\right) \approx 35°$.
$\cos\angle P = \frac{7{,}5}{10} \Rightarrow \angle P = \cos^{-1}\left(\frac{7{,}5}{10}\right) \approx 41°$.



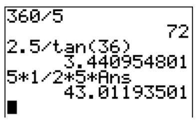


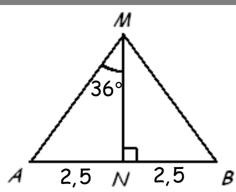
2 $\cos(38°) = \frac{AC}{17} \Rightarrow AC = 17\cos(38°) \approx 13{,}4$.
$\tan(55°) = \frac{DF}{5} \Rightarrow DF = 5\tan(55°) \approx 7{,}1$.
$\sin(40°) = \frac{7}{HG} \Rightarrow HG = \frac{7}{\sin(40°)} \approx 10{,}9$.
$\sin(60°) = \frac{MN}{17} \Rightarrow MN = KL = 17\sin(60°) \approx 14{,}7$.
$\tan(75°) = \frac{RS}{3} \Rightarrow RS = 3\tan(75°) \approx 11{,}2$.





7 $\angle AMB = \frac{360°}{5} = 72° \Rightarrow \angle AMN = 36°$ en $\tan(36°) = \frac{2{,}5}{MN} \Rightarrow MN = \frac{2{,}5}{\tan(36°)}$.
$O(ABCDE) = 5 \cdot \frac{1}{2} \cdot AB \cdot MN = 5 \cdot \frac{1}{2} \cdot 5 \cdot \frac{2{,}5}{\tan(36°)} \approx 43{,}01$.

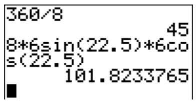


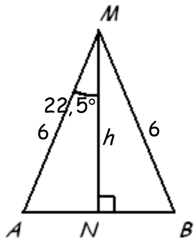


8 $\angle AMB = \frac{360^\circ}{8} = 45^\circ$.

$\cos(22{,}5^\circ) = \frac{h}{6} \Rightarrow h = 6\cos(22{,}5^\circ)$ en $\sin(22{,}5^\circ) = \frac{\frac{1}{2}AB}{6} \Rightarrow \frac{1}{2}AB = 6\sin(22{,}5^\circ)$.

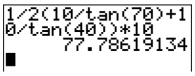
$O(ABCDEFGH) = 8 \cdot \frac{1}{2} \cdot AB \cdot h = 8 \cdot 6\sin(22{,}5^\circ) \cdot 6\cos(22{,}5^\circ) \approx 101{,}82$.

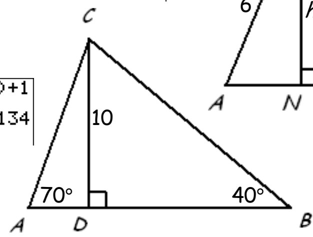




9 $\tan(70^\circ) = \frac{10}{AD} \Rightarrow AD = \frac{10}{\tan(70^\circ)}$ en $\tan(40^\circ) = \frac{10}{DB} \Rightarrow DB = \frac{10}{\tan(40^\circ)}$.

$O(\Delta ABC) = \frac{1}{2} \cdot (AD + DB) \cdot h = \frac{1}{2} \cdot (\frac{10}{\tan(70^\circ)} + \frac{10}{\tan(40^\circ)}) \cdot 10 \approx 77{,}8$.

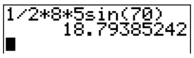




10a Zie de figuur hiernaast.

10b ΔADC: $\sin(70^\circ) = \frac{h}{5} \Rightarrow h = 5\sin(70^\circ)$.

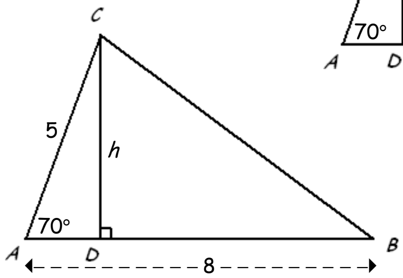
$O(\Delta ABC) = \frac{1}{2} \cdot 8 \cdot 5\sin(70^\circ) \approx 18{,}8$.



10c Teken de hoogtelijn CD en noem deze h.

ΔADC: $\sin\angle A = \frac{h}{b} \Rightarrow h = b\sin\angle A$.

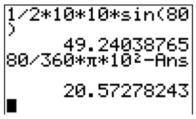
$O(\Delta ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot c \cdot b\sin\angle A = \frac{1}{2}bc\sin\angle A$.



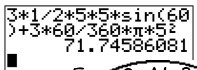
ONTHOUD: de oppervlakte van een driehoek is $\frac{1}{2} \times$ zijde $\times$ zijde $\times$ de sinus van de ingesloten hoek (door deze twee zijden)

11a $O(\Delta ABM) = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin(80^\circ) = 50 \cdot \sin(80^\circ) \approx 49{,}24$. (de formule van opgave 10c)

11b $O(\text{segment}) = O(\text{sector } ABM) - O(\Delta ABM) = \frac{80}{360} \cdot \pi \cdot 10^2 - 50\sin(80^\circ) \approx 20{,}57$.



12 $O(ABCDEF) = 3 \cdot O(\Delta ABM) + 3 \cdot O(\text{sector } BCM) = 3 \cdot \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(60^\circ) + 3 \cdot \frac{60}{360} \cdot \pi \cdot 5^2 \approx 71{,}75$.

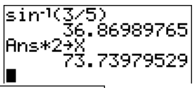


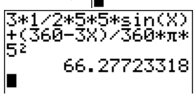
13 ΔMDN: $\sin\angle DMN = \frac{3}{5} \Rightarrow \angle DMN = \sin^{-1}\left(\frac{3}{5}\right)$.

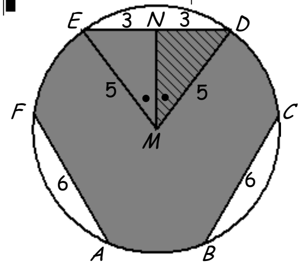
$\angle DME = \angle AMF = \angle BMC = 2 \cdot \angle DMN = 2 \cdot \sin^{-1}\left(\frac{3}{5}\right)$ (opslaan in X).

$O(ABCDEF) = 3 \cdot O(\Delta MDE) + O(3 \text{ sectoren})$

$= 3 \cdot \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(X) + \frac{360 - 3X}{360} \cdot \pi \cdot 5^2 \approx 66{,}3$.







14 ΔMRP: $\sin\angle RMP = \frac{2{,}5}{4} \Rightarrow \angle RMP = \sin^{-1}\left(\frac{2{,}5}{4}\right)$.

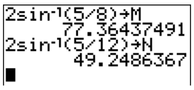
$\angle QMP = 2\angle RMP = 2\sin^{-1}\left(\frac{2{,}5}{4}\right)$ (opslaan in M).

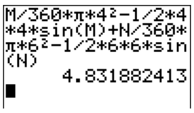
ΔNRP: $\sin\angle RNP = \frac{2{,}5}{6} \Rightarrow \angle RNP = \sin^{-1}\left(\frac{2{,}5}{6}\right)$.

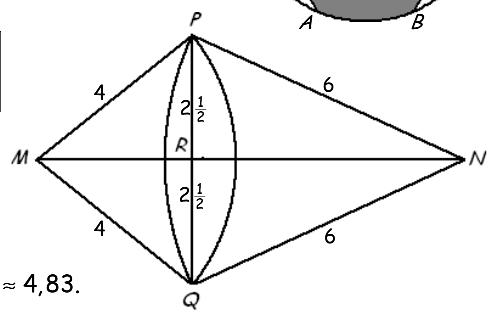
$\angle QNP = 2\angle RNP = 2\sin^{-1}\left(\frac{2{,}5}{6}\right)$ (opslaan in N).

$O(\text{gebied}) = O(\text{sector } MPQ) - O(\Delta MPQ) + O(\text{sector } NPQ) - O(\Delta NPQ)$

$= \frac{M}{360} \cdot \pi \cdot 4^2 - \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin(M) + \frac{N}{360} \cdot \pi \cdot 6^2 - \frac{1}{2} \cdot 6 \cdot 4 \cdot \sin(N) \approx 4{,}83$.



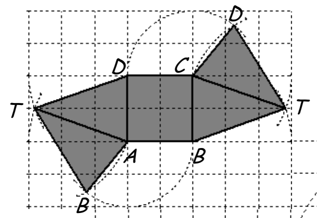




15a De figuren a, b en d kunnen de uitslag van een kubus zijn.

16a De figuren a, b en d kunnen de uitslag van een kubus zijn.

17 Zie hiernaast de uitslag van de piramide in figuur 2.31.



18a ΔABT is een vergroting van ΔEFT (snavelfiguur).

Omdat $AB = 4$ en $EF = 2$ is de vergrotingsfactor $\frac{4}{2} = 2$.

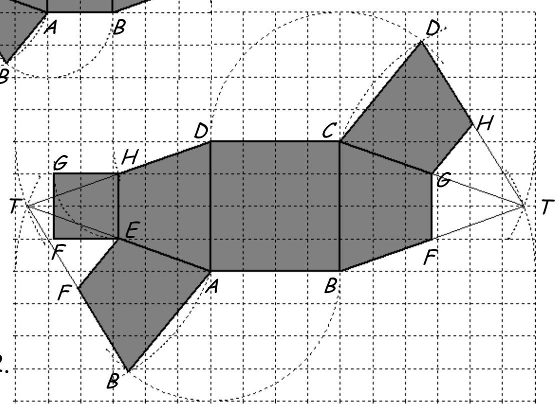
ΔABT	$AB = 4$	$AT = 3 + x$	...
ΔEFT	$EF = 2$	$ET = x$	...

×2

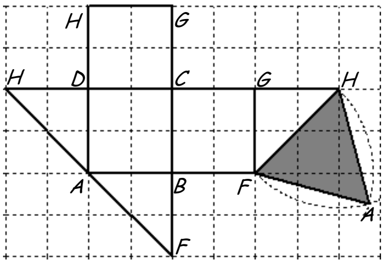
$4x = 2(3 + x) \Rightarrow 4x = 6 + 2x \Rightarrow 2x = 6 \Rightarrow x = 3$.

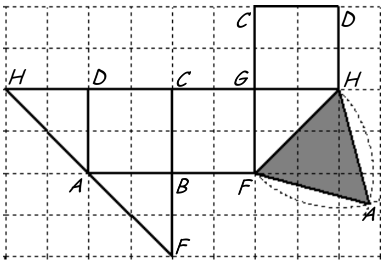
$AT = 3 + x \Rightarrow AT = 6$.

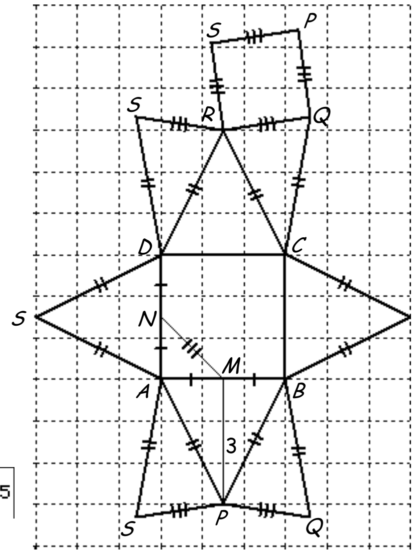
18b Zie hiernaast een uitslag van de afgeknotte piramide in figuur 2.32.



19 Zie hieronder twee mogelijke uitslagen van het lichaam in figuur 2.33.







20 $PM = 3 \Rightarrow \Delta ABP$, ΔBCQ, ΔCDR en ΔADS in de uitslag te tekenen.
PS in het bovenvlak is even lang als MN in het grondvlak.
Zie hiernaast een uitslag van het lichaam in figuur 2.34.

21 De lengte van de cilindermantels (de rechthoeken) is 5 (cm).
Dus de omtrek van de cirkels is $2\pi r = 5 \Rightarrow r = \frac{5}{2\pi} \approx 0{,}8$ (cm).
De straal van de blauwe cirkels is 0,8 (cm) $\Rightarrow$ het is figuur 2.35b.

```
5/(2π)
      .7957747155
```

22a $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$
$\cancel{2\pi} \cdot r = \frac{90}{360} \cdot \cancel{2\pi} \cdot 3$
$r = \frac{90}{360} \cdot 3 = \frac{3}{4}$ (cm).

```
90/360*3
              .75
```

22b $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$
$2\pi \cdot r = \frac{210}{360} \cdot 2\pi \cdot 2{,}5$
$r = \frac{210}{360} \cdot 2{,}5 \approx 1{,}5$ (cm).

```
210/360*2.5
      1.458333333
Ans▸Frac
            35/24
```

22c $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$
$2\pi \cdot r = \frac{300}{360} \cdot 2\pi \cdot 2$
$r = \frac{300}{360} \cdot 2 \approx 1{,}7$ (cm).

```
300/360*2
      1.666666667
Ans▸Frac
              5/3
```

23 $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$
$2\pi \cdot r = \frac{p}{360} \cdot 2\pi \cdot R$
$r = \frac{p}{360} \cdot R$.

24 $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$ (met middelpuntshoek $p°$)
$\cancel{2\pi} \cdot 3 = \frac{p}{360} \cdot \cancel{2\pi} \cdot 5$
$p = \frac{3}{5} \cdot 360 = 216 \Rightarrow$ de middelpuntshoek is 216°.

```
3/5*360
              216
```

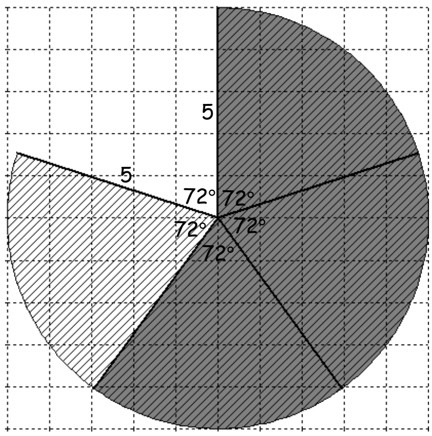
25a $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$ (met middelpuntshoek $p°$)
$\cancel{2\pi} \cdot 3 = \frac{p}{360} \cdot \cancel{2\pi} \cdot 5$ ($R^2 = 3^2 + 4^2 = 9 + 16 = 25 \Rightarrow R = 5$)
$p = \frac{3}{5} \cdot 360 = 216 \Rightarrow$ middelpuntshoek is 216°.

```
3/5*360
              216
```

De uitslag is $\frac{3}{5}$ deel van een cirkel met straal 5.

```
1/5*360
               72
```

(zie het grijs gemarkeerde deel hiernaast)



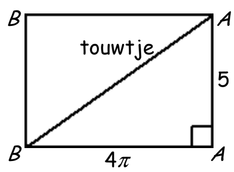
25b $\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$ (met middelpuntshoek $p°$)
$\cancel{2\pi} \cdot 4 = \frac{p}{360} \cdot \cancel{2\pi} \cdot 5$ ($R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$)
$p = \frac{4}{5} \cdot 360 = 288 \Rightarrow$ middelpuntshoek is 288°.

```
4/5*360
              288
```

De uitslag is $\frac{4}{5}$ deel van een cirkel met straal 5.
(zie het gearceerde deel hiernaast)

26a De uitslag van de cilindermantel is een rechthoek van $2\pi \cdot 2 = 4\pi$ cm bij 5 cm.

26b Zie een schets van de rechthoek hiernaast, de diagonaal stelt het touwtje voor.



26c De stelling van Pythagoras geeft $AB = \sqrt{(4\pi)^2 + 5^2} \approx 13{,}5$ (cm).

```
√((4π)²+5²)
      13.52455805
```

27 Over een hoogte van $6 - 1 = 5$ meter is het touw precies vier keer strak om de mast gewonden,
dus over een hoogte van 125 cm is het touw precies één keer strak om de mast gewonden.
De diagonaal van de mantel van de cilinder met $h = 125$ (cm) en $r = 4{,}5$ (de diameter van de mast is 9 cm)
is $\sqrt{(2\pi \cdot 4{,}5)^2 + 125^2}$ (de diagonaal in een rechthoek van $2\pi \cdot 4{,}5$ cm bij 125 cm) $\Rightarrow$ het touw is $4 \cdot \text{Ans} \approx 513$ cm lang.

```
√((2π*4.5)²+125²
)
      128.1578634
Ans*4
      512.6314537
```

28 $O(\text{cilinder}) = 2 \cdot O(\text{grondcirkel}) + O(\text{cilindermantel})$
$= 2 \cdot \pi r^2 + 2\pi r \cdot h = 2 \cdot \pi \cdot 3^2 + 2\pi \cdot 3 \cdot 4 = 18\pi + 24\pi = 42\pi \approx 132$ (cm²).

```
42π
      131.9468915
```

29a omtrek$_{\text{grondcirkel}}$ = lengte$_{\text{boog}}$ (met middelpuntshoek $p°$)

$\cancel{2\pi}\cdot 3=\frac{p}{360}\cdot\cancel{2\pi}\cdot 5$ ($R^2=3^2+4^2=25\Rightarrow R=5$). Dus $p=\frac{3}{5}\cdot 360=216\Rightarrow$ middelpuntshoek is 216°.

```
3/5*360
                216
```

29b $O(\text{kegelmantel})=\frac{216}{360}\cdot\pi\cdot 5^2=15\pi\approx 47$ (cm²).

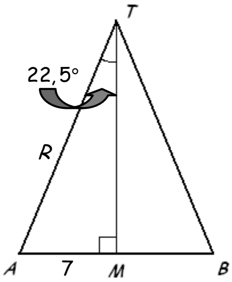
29c $O(\text{kegel})=O(\text{grondcirkel})+O(\text{kegelmantel})=\pi\cdot 3^2+15\pi=9\pi+15\pi=24\pi\approx 75$ (cm²).

```
216/360*5²
                 15
Ans*π
         47.1238898
24π
        75.39822369
```

30 $\sin(22{,}5°)=\frac{7}{R}\Rightarrow R=\frac{7}{\sin(22{,}5°)}$

$O(\text{kegel})=O(\text{grondcirkel})+O(\text{kegelmantel})=\pi r^2+\pi rR\approx 556{,}2$ (cm²).

```
7/sin(22.5)→R
        18.29188151
π*7²+π*7*R
        556.197524
```



$P(\text{cirkel})=2\pi r$
$O(\text{cirkel})=\pi r^2$
$O(\text{cilindermantel})=2\pi rh$
$O(\text{kegelmantel})=\pi rR$

31 $O(\text{grondcirkel})=\pi r^2=100\Rightarrow r^2=\frac{100}{\pi}\Rightarrow r=\sqrt{\frac{100}{\pi}}$.

$R^2=r^2+h^2=\frac{100}{\pi}+10=\frac{100}{\pi}+100\Rightarrow R=\sqrt{\frac{100}{\pi}+100}$.

$O(\text{kegelmantel})=\pi rR\approx 203{,}5$ (cm²).

```
π*√(100/π)*√(100
/π+100)
        203.5090331
```

32 $O(\text{grondcirkel})=\pi r^2=50\Rightarrow r^2=\frac{50}{\pi}\Rightarrow r=\sqrt{\frac{50}{\pi}}$.

$O(\text{kegelmantel})=\pi rR=75\Rightarrow R=\frac{75}{\pi r}$.

Noem de halve tophoek α dan $\sin\alpha=\frac{r}{R}\Rightarrow\alpha=\sin^{-1}(\frac{r}{R})\Rightarrow$ tophoek $2\alpha\approx 84°$.

```
50/π
        15.91549431
√(Ans)→X
        3.989422804
75/(πX)→R
        5.984134206
```

```
sin⁻¹(X/R)
        41.8103149
Ans*2
        83.62062979
```

33a $\Delta MAT \backsim \Delta NBT$ (snavelfiguur).

$5x=2(6+x)\Rightarrow 5x=12+2x\Rightarrow 3x=12\Rightarrow NT=x=4$ (en $MT=10$).

ΔMAT	$MA=5$	$MT=6+x$	...
ΔNBT	$NB=2$	$NT=x$	...

$\times 2\frac{1}{2}$

33b Van de kegel met top T en straal grondcirkel $r=AM=5$ is

$R^2=r^2+h^2=5^2+10^2=25+100=125\Rightarrow R=\sqrt{125}\Rightarrow O(\text{kegelmantel})=\pi rR=\pi\cdot 5\cdot\sqrt{125}\approx 175{,}62$.

33c Van de kegel met top T en straal grondcirkel $r=NB=2$ is

$R^2=r^2+h^2=2^2+4^2=4+16=20\Rightarrow R=\sqrt{20}\Rightarrow O(\text{kegelmantel})=\pi rR=\pi\cdot 2\cdot\sqrt{20}$.

Dus $O(\text{mantel van afgeknotte kegel})=5\pi\sqrt{125}-2\pi\sqrt{20}\approx 147{,}52$.

```
π*5*√(125)
        175.6203683
Ans-π*2*√(20)
        147.5211094
Ans+π*5²+π*2²
        238.6272963
```

33d $O(\text{afgeknotte kegel})=O(\text{mantel})+O(\text{gondcirkel})+O(\text{topcirkel})=5\pi\sqrt{125}-2\pi\sqrt{20}+\pi\cdot 5^2+\pi\cdot 2^2\approx 238{,}63$.

34 $\Delta MAT \backsim \Delta NBT$ (snavelfiguur). (zie voor de letters figuur 2.51)

$10x=4(11+x)\Rightarrow 10x=44+4x\Rightarrow 6x=44\Rightarrow NT=x=\frac{22}{3}$ (en $MT=\frac{55}{3}$).

ΔMAT	$MA=10$	$MT=11+x$	...
ΔNBT	$NB=4$	$NT=x$	...

$\times 2\frac{1}{2}$

Van de kegel met top T en straal grondcirkel $r=AM=10$ (cm) is

$R^2=r^2+h^2=10^2+(\frac{55}{3})^2=\frac{3925}{9}\Rightarrow R=\sqrt{\frac{3925}{9}}$ (cm) $\Rightarrow O(\text{kegelmantel})=\pi rR=\pi\cdot 10\cdot\sqrt{\frac{3925}{9}}$ (cm²).

Van de kegel met top T en straal grondcirkel $r=NB=4$ (cm) is

$R^2=r^2+h^2=4^2+(\frac{22}{3})^2=\frac{628}{9}\Rightarrow R=\sqrt{\frac{628}{9}}$ (cm) $\Rightarrow O(\text{kegelmantel})=\pi rR=\pi\cdot 4\cdot\sqrt{\frac{628}{9}}$ (cm²).

$O(\text{lampenkapje})=10\pi\sqrt{\frac{3925}{9}}-4\pi\sqrt{\frac{628}{9}}\approx 551$ (cm²).

```
10²+(55/3)²▶Frac
             3925/9
4²+(22/3)²▶Frac
              628/9
```

```
10π√(3925/9)-4π√
(628/9)
        551.0966037
```

35 $O(\text{linker cilinder})=2\cdot O(\text{gondcirkel})+O(\text{cilindermantel})=2\cdot\pi r^2+2\pi r\cdot h=2\cdot\pi\cdot 4^2+2\pi\cdot 4\cdot 10=32\pi+80\pi=112\pi$.

$O(\text{rechter cilinder})=2\cdot O(\text{gondcirkel})+O(\text{cilindermantel})=2\cdot\pi r^2+2\pi r\cdot h=2\cdot\pi\cdot 2^2+2\pi\cdot 2\cdot h=8\pi+4\pi h$.

$8\pi+4\pi h=112\pi\Rightarrow 4\pi h=104\pi$ (delen door π) $\Rightarrow 4h=104\Rightarrow h=26$.

36 $O(\text{skatebaan})=O(\text{rechthoek})+\frac{1}{2}\cdot O(\text{cilindermantel})$

$=l\cdot b+\frac{1}{2}\cdot 2\pi r\cdot h=(12-2\cdot 3)\cdot 6+\frac{1}{2}\cdot 2\pi\cdot 3\cdot 6=6\cdot 6+\pi\cdot 18=36+18\pi$ (m²).

De materiaalkosten van de staalplaten (€1,75/dm² $\Rightarrow$ €175/m²) is $(36+18\pi)\cdot 175\approx 16196$ (€)

```
36+18π
        92.54866776
Ans*100*1.75
        16196.01686
```

37 $O(\text{mantel figuur 2.55a})=\frac{1}{2}\cdot O(\text{cilindermantel figuur 2.55b})=\frac{1}{2}\cdot 2\pi r\cdot h=\pi\cdot 12\cdot 20=240\pi$ (cm²).

```
240π
        753.9822369
```

38 $O(\text{twee bolletjes met straal } r)=O(\text{bol met straal 5})$

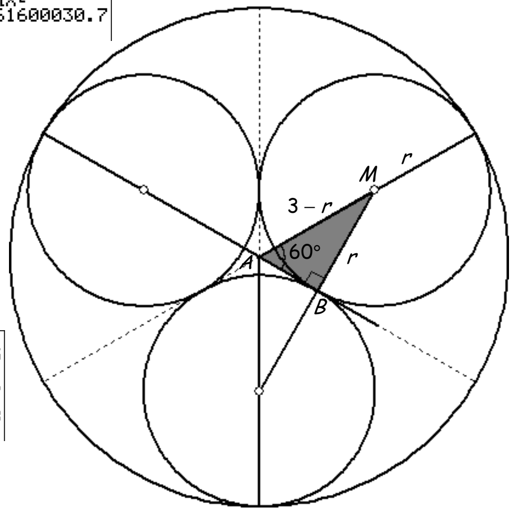
$2\cdot 4\pi r^2=4\pi\cdot 5^2\Rightarrow 8\pi r^2=100\pi\Rightarrow r^2=12{,}5\Rightarrow r=\sqrt{12{,}5}\approx 3{,}54$.

```
√(12.5)
        3.535533906
```

$P(\text{cirkel})=2\pi r$
$O(\text{cirkel})=\pi r^2$
$O(\text{bol})=4\pi r^2$

39 $P(\text{aardbol}) = 2\pi r = 40\,000$ (km) $\Rightarrow r = \frac{40000}{2\pi} = \frac{20000}{\pi}$ (km).
$O(\text{oceanen}) = 0{,}71 \times O(\text{aarbol}) = 0{,}71 \times 4\pi r^2 \approx 361\,600\,000$ (km^2).

```
20000/π→X
          6366.197724
0.71*4πX²
          361600030.7
```

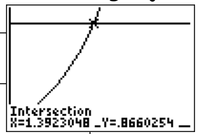


40a $O(\text{cilindermantel}) = 2\pi r \cdot h = 2\pi \cdot 3 \cdot 12 = 72\pi$ (cm^2).
(in figuur 2.59a zie je dat $h = 4r = 12 \Rightarrow r = 3$)
$O(\text{bal}) = 4\pi r^2 = 4\pi \cdot 3^2 = 4\pi \cdot 9 = 36\pi$ (cm^2). Dus Linda heeft gelijk.

40b Zie de figuur hiernaast.
In ΔABM is $\sin(60°) = \frac{r}{3-r}$.
Intersect geeft $r \approx 1{,}39$ (cm).

```
Plot1 Plot2 Plot3
\Y1■X/(3-X)
\Y2■sin(60°)
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=3
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
 Xres=1
```



40c Vier lagen van drie knikkers.
Als de lagen precies op elkaar liggen komen ze $8r < 8 \cdot 1{,}5 = 12$ cm hoog.
(als de lagen in de kuiltjes liggen komen ze minder hoog)

```
X
          1.392304845
8X
          11.13843876
48πX²
          292.3208408
```

40d $O(\text{twaalf knikkers}) = 12 \cdot O(\text{knikker}) = 12 \cdot 4\pi r^2 = 48\pi r^2 \approx 292$ (cm^2).

40e $O(\text{twee ballen}) = 2 \cdot 36\pi = 72\pi$ (cm^2).
$O(\text{twaalf knikkers}) \div O(\text{twee ballen}) = \frac{48\pi r^2}{72\pi} \approx 1{,}29$.

```
48πX²/(72π)
          1.292341855
```

41a $I(\text{kegel}) = \frac{1}{3} \cdot I(\text{cilinder}) = \frac{1}{3} \cdot \pi r^2 h = \frac{1}{3}\pi r^2 h$.

41b $I(\text{bol}) = 4 \cdot I(\text{kegel}) = 4 \cdot \frac{1}{3}\pi r^2 h$ (in figuur 2.64 is $h = r$) $= \frac{4}{3}\pi r^3$.

42 Er passen 3 ballen in de doos $\Rightarrow h = 6r$. Dus $I(\text{doos}) = G \cdot h = \pi r^2 \cdot 6r = 6\pi r^3$ ($= 100\%$).
$I(\text{drie ballen}) = 3 \cdot I(\text{bal}) = 3 \cdot \frac{4}{3}\pi r^3 = 4\pi r^3$. Het percentage is dus $\frac{4\pi r^3}{6\pi r^3} \times 100\% = \frac{2}{3} \times 100\% \approx 66{,}7\%$.

```
2/3
          .6666666667
Ans*100
          66.66666667
```

43a $G = \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin(60°) = 18\sin(60°)$.
$I(\text{prisma}) = G \cdot h = 18\sin(60°) \cdot 8 = 144\sin(60°) \approx 124{,}7$.

```
1/2*6*6*sin(60)
          15.58845727
Ans*8
          124.7076581
```

```
13²-5²
                  144
√(Ans)
                   12
```

43b $G = \frac{1}{2} \cdot 10 \cdot 12 = 60$. (Zij M het midden van HK, dan $HM^2 + GM^2 = HG^2 \Rightarrow 5^2 + GM^2 = 13^2 \Rightarrow GM = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$)
$I(\text{piramide}) = \frac{1}{3}Gh = \frac{1}{3} \cdot 60 \cdot 9 = 20 \cdot 9 = 180$.

43c $I(\text{kegel}) = \frac{1}{3}Gh = \frac{1}{3} \cdot \pi \cdot 2^2 \cdot \sqrt{12} \approx 14{,}5$. ($2^2 + h^2 = 4^2 \Rightarrow h = \sqrt{4^2 - 2^2} = \sqrt{12}$)

```
4²-2²
                   12
1/3π*4*√(12)
          14.51039491
```

44 $I(\text{piramide}) = \frac{1}{3}Gh = \frac{1}{3} \cdot (2 \cdot 2 - \frac{1}{2} \cdot 1 \cdot 1) \cdot \sqrt{3} = \frac{1}{3} \cdot 3\frac{1}{2} \cdot \sqrt{3} \approx 2{,}0$ (cm^3). ($1^2 + h^2 = 2^2 \Rightarrow h = \sqrt{2^2 - 1^2} = \sqrt{4}$)

```
1/3*3.5*√(3)
          2.020725942
```

45 Noem P het snijpunt van AE en GH; Q het snijpunt van IH en CF (B is het snijpunt van AE en CF).
$I(\text{woning}) = I(T.ABCD) - I(EIJ.PHK) - I(K.PBQH) - I(QHK.FGL) = I(\text{piramide}) - I(\text{prisma}) - I(\text{piramide}) - I(\text{prisma})$
$= \frac{1}{3} \cdot 16 \cdot 16 \cdot 12 - \frac{1}{2} \cdot 4 \cdot 6 \cdot 8 - \frac{1}{3} \cdot 4 \cdot 4 \cdot 6 - \frac{1}{2} \cdot 4 \cdot 6 \cdot 4$
$= 1024 - 96 - 32 - 48 = 848$ (m^3).

```
16*16*12/3
                 1024
4*6*8/2
                   96
4*4*6/3
                   32
```

```
4*6*4/2
                   48
1024-96-32-48
                  848
```

46 Het huis bestaat uit twee prisma's ieder met een vijfhoek als grondvlak en een piramide die de zolders verbindt.
$I(\text{huis}) = I(\text{prisma op de voorgond}) + I(\text{prisma}) + I(\text{piramide op de achtergrond}) = G_1 \cdot h_1 + G_2 \cdot h_2 + \frac{1}{3} \cdot G_3 \cdot h_3$
$= (4 \cdot 3 + \frac{1}{2} \cdot 4 \cdot 4) \cdot 5 + (6 \cdot 3 + \frac{1}{2} \cdot 6 \cdot 4) \cdot 14 + \frac{1}{3} \cdot \frac{1}{2} \cdot 4 \cdot 4 \cdot 3$
$= (12 + 8) \cdot 5 + (18 + 12) \cdot 14 + 2 \cdot 4 = 20 \cdot 5 + 30 \cdot 14 + 8 = 100 + 420 + 8 = 528$ (m^3).

```
(4*3+1/2*4*4)*5+
(6*3+1/2*6*4)*14
+1/3*1/2*4*4*3
                  528
```

47 Het grondvlak en zeshoek met zijden van $1{,}5 \cdot 5 = 7{,}5$ cm; de hoogte van de doos is $2{,}75 \cdot 5 = 13{,}75$ cm.
$I(\text{doos}) = I(\text{prisma}) = G \cdot h = 6 \cdot \frac{1}{2} \cdot 7{,}5 \cdot 7{,}5 \cdot \sin(60°) \cdot 13{,}75 \approx 2\,009$ (cm^3).
(G is de oppervlakte van 6 gelijkzijdige diehoeken met zijden van 7,5 cm)

```
2.75*5
                13.75
6*1/2*7.5*7.5*si
n(60)*13.75
           2009.44957
```

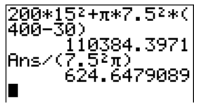
48 $I(\text{bol}) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi$ (cm^3).
$I(\text{kegel}) = \frac{1}{3}Gh = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \cdot 10 = \frac{10}{3}\pi r^2 = \frac{500}{3}\pi$ (cm^3) $\Rightarrow r^2 = 50 \Rightarrow r_{kegel} = \sqrt{50} \approx 7{,}1$ (cm).
$I(\text{cilinder}) = Gh = \pi r^2 h = \pi r^2 \cdot 10 = 10\pi r^2 = \frac{500}{3}\pi$ (cm^3) $\Rightarrow r^2 = \frac{50}{3} \Rightarrow r_{cilinder} = \sqrt{\frac{50}{3}} \approx 4{,}1$ (cm).

```
√(50)
          7.071067812
√(50/3)
          4.082482905
```

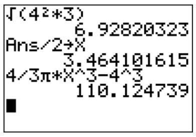
49 $I(\text{bol}) = \frac{4}{3}\pi r^3$; $I(\text{kegel}) = \frac{1}{3}Gh = \frac{1}{3}\pi r^2 \cdot 2r = \frac{2}{3}\pi r^3$ en $I(\text{cilinder}) = Gh = \pi r^2 \cdot 2r = 2\pi r^3$.
$I(\text{kegel}) : I(\text{bol}) : I(\text{cilinder}) = \frac{2}{3}\pi r^3 : \frac{4}{3}\pi r^3 : 2\pi r^3 = \frac{2}{3} : \frac{4}{3} : \frac{6}{3} = 2 : 4 : 6 = 1 : 2 : 3$.

50a $I(\text{systeem}) = I(\text{balk}) + I(\text{buizen}) = 200 \cdot 15 \cdot 15 + \pi \cdot 7{,}5^2 \cdot (2 \cdot 200 - 2 \cdot 15) \approx 110\,384$ (cm^3).

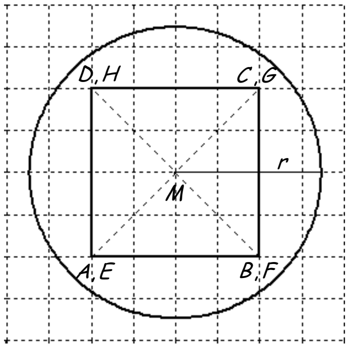
50b $I(\text{extra buis}) = \pi \cdot 7{,}5^2 \cdot h = I(\text{systeem}) \Rightarrow h = \frac{I(\text{systeem})}{7{,}5^2\pi} \approx 625$ (cm).



51a Bereken lichaamsdiagonaal AG in de kubus met ribbe $AB = 4$ (cm).
$AG = 2r = \sqrt{4^2 + 4^2 + 4^2} = \sqrt{3 \cdot 4^2} = 4\sqrt{3}$ (cm) $\Rightarrow r = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$ (cm).
$I(\text{bol}) - I(\text{kubus}) = \frac{4}{3}\pi r^3 - 4^3 = \frac{4}{3}\pi \cdot (2\sqrt{3})^3 - 4^3 \approx 110{,}12$ (cm).

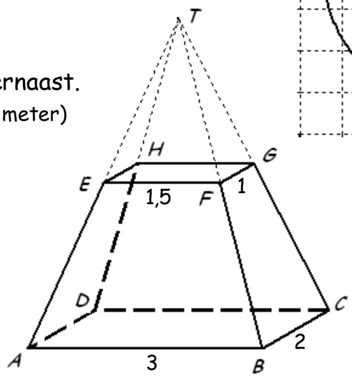


51b Teken het bovenvlak van de kubus met daar omheen de cirkel met straal $r = 2\sqrt{3} \approx 3{,}5$ (cm). Zie hiernaast.



52a Zet het karretje "op z'n kop" (en sloop het onderstel). Zie hiernaast.
(omdat $EF = \frac{1}{2}AB$ en $FG = \frac{1}{2}BC$ is de hoogte van de hele piramide 4 meter)

52b $I(\text{kar}) = \frac{1}{3} \cdot \cancel{3} \cdot 2 \cdot 4 - \cancel{\frac{1}{3} \cdot 1{,}5} \cdot 1 \cdot \cancel{2} = 8 - 1 = 7$ (m^3).

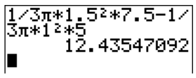


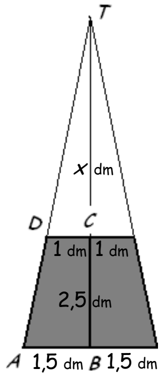
53 De emmer heeft de vorm van een afgeknotte piramide.
$\Delta ABT \backsim \Delta DCT$ (snavelfiguur).

ΔABT	$AB = 1{,}5$	$BT = x + 2{,}5$	...
ΔDCT	$DC = 1$	$CT = x$	...

$\times 1\frac{1}{2}$

$1{,}5x = x + 2{,}5 \Rightarrow 0{,}5x = 2{,}5 \Rightarrow x = CT = 5$ dm (en $BT = 7{,}5$ dm).
$I(\text{emmer}) = \frac{1}{3} \cdot \pi \cdot 1{,}5^2 \cdot 7{,}5 - \frac{1}{3} \cdot \pi \cdot 1^2 \cdot 5 \approx 12{,}4$ (dm^3 = liter).

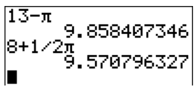




Diagnostische toets

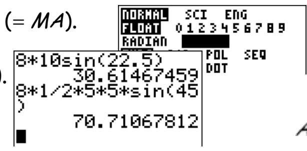
D1a $O = 4 \cdot 4 - \frac{1}{2} \cdot 2 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 1 - \pi \cdot 1^2 = 16 - 2 - 1 - \pi = 13 - \pi \approx 9{,}86$ cm^2 = 986 mm^2.

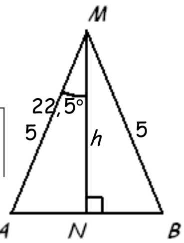
D1b $O = 4 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 1 - \frac{1}{2} \cdot 2 \cdot 1 + \frac{1}{2} \cdot \pi \cdot 1^2 = 8 + 1 - 1 + \frac{1}{2}\pi = 8 + \frac{1}{2}\pi \approx 9{,}57$ cm^2 = 957 mm^2.



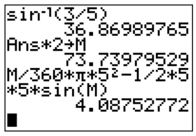
D2a $\angle AMB = \frac{360^\circ}{8} = 45^\circ \Rightarrow \angle MAB + \angle MBA = 180^\circ - 45^\circ = 135^\circ$
$\angle MAB = \angle MBA \Rightarrow 2\angle MBA = 135^\circ = \angle ABC$.

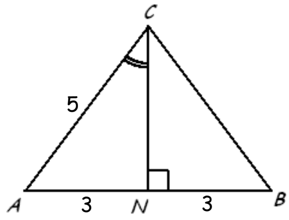
D2b De omtrek van omgeschreven cirkel is $2\pi r = 10\pi$ (gegeven) $\Rightarrow r = \frac{10\pi}{2\pi} = 5$ ($= MA$).
$\sin\angle AMN = \sin(22{,}5^\circ) = \frac{\frac{1}{2}AB}{5} \Rightarrow \frac{1}{2}AB = 5\sin(22{,}5^\circ) \Rightarrow AB = 10\sin(22{,}5^\circ)$.
Omtrek($ABCDEFGH$) $= 8 \cdot AB = 8 \cdot 10\sin(22{,}5^\circ) \approx 30{,}6$.
$O(ABCDEFGH) = 8 \cdot \frac{1}{2} \cdot MA \cdot MB \cdot \sin\angle AMB = 8 \cdot \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(45^\circ) \approx 70{,}7$.



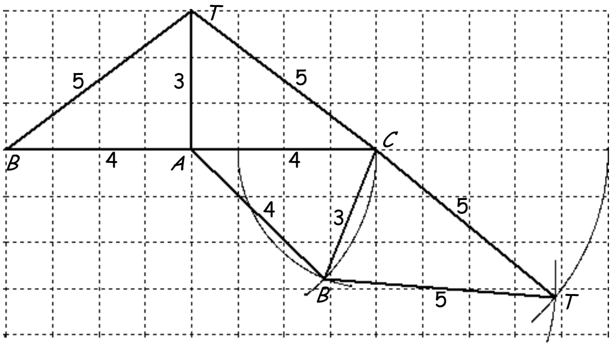


D3 $\sin\angle AMN = \frac{3}{5} \Rightarrow \angle AMN \approx 36{,}9°$. $\angle AMB = 2\angle AMN$ (opslaan in M).
$O(\text{segment}) = O(\text{sector}) - O(\Delta AMB) = \frac{M}{360} \cdot \pi \cdot 5^2 - \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin(M) \approx 4{,}09$.

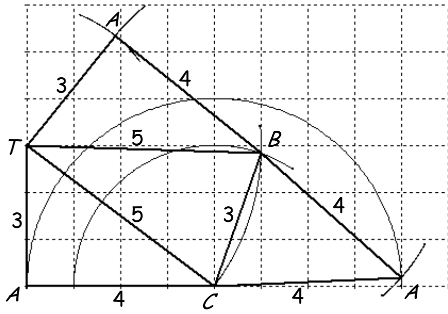




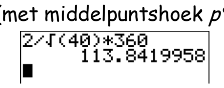
D4a $BT = CT = \sqrt{4^2 + 3^2} = 5$. Zie hieronder de uitslag.



D4b Zie de uitslag hiernaast.

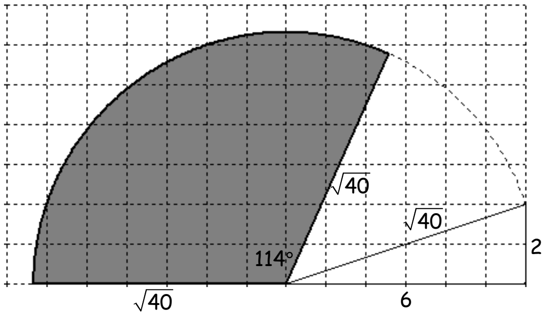


D5a $R^2 = 2^2 + 6^2 = 4 + 36 = 40 \Rightarrow R = \sqrt{40}$.
$\text{omtrek}_{\text{grondcirkel}} = \text{lengte}_{\text{boog}}$ (met middelpuntshoek $p°$)
$\cancel{2\pi} \cdot 2 = \frac{p}{360} \cdot \cancel{2\pi} \cdot \sqrt{40}$

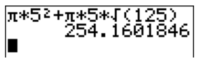


$p = \frac{2}{\sqrt{40}} \cdot 360 \approx 114 \Rightarrow$ middelpuntshoek is (ongeveer) $114°$.

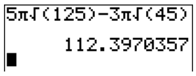
D5b De uitslag van de kegelmantel zie je hiernaast.
(haal $R = \sqrt{40}$ uit een driehoek met rechthoekszijden 2 en 6)



D6a $R^2 = r^2 + h^2 = 5^2 + 10^2 = 25 + 100 = 125 \Rightarrow R = \sqrt{125}$
$O(\text{kegel}) = O(\text{grondcirkel}) + O(\text{kegelmantel}) = \pi r^2 + \pi rR = \pi \cdot 5^2 + \pi \cdot 5 \cdot \sqrt{125} \approx 254{,}2$.



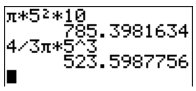
D6b Van de topkegel met $h = 10 - 4 = 6$ en straal grondcirkel $r = \frac{6}{10} \cdot 5 = 3$ (snavelfiguur) is
$R^2 = r^2 + h^2 = 3^2 + 6^2 = 9 + 36 = 45 \Rightarrow R = \sqrt{45} \Rightarrow O(\text{kegelmantel}) = \pi rR = \pi \cdot 3 \cdot \sqrt{45}$.
$O(\text{mantel van afgeknotte kegel}) = 5\pi\sqrt{125} - 3\pi\sqrt{45} \approx 112{,}4$.



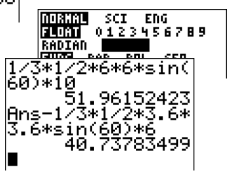
D7a $O(\text{cilinder}) = 2 \cdot O(\text{grondcirkel}) + O(\text{cilindermantel}) = 2 \cdot \pi r^2 + 2\pi rh = 2 \cdot \pi \cdot 4^2 + 2\pi \cdot 4 \cdot 8 = 32\pi + 64\pi = 96\pi$.
$O(\text{bol}) = 4\pi r^2 = 4\pi \cdot 4^2 = 64\pi$ (= 100%) en $O(\text{cilinder}) = \frac{96\pi}{64\pi} \times 100\% = \frac{3}{2} \times 100\% = 150\%$.
Dus O(cilinder) is 50% groter dan O(bol).

D7b $O(\text{cilinder}) = 2 \cdot O(\text{grondcirkel}) + O(\text{cilindermantel}) = 2 \cdot \pi r^2 + 2\pi rh = 2 \cdot \pi \cdot r^2 + 2\pi \cdot r \cdot 2r = 2\pi r^2 + 4\pi r^2 = 6\pi r^2$.
$O(\text{bol}) = 4\pi r^2$ (= 100%) en $O(\text{cilinder}) = \frac{6\pi r^2}{4\pi r^2} \times 100\% = \frac{3}{2} \times 100\% = 150\%$.
Dus O(cilinder) is steeds 50% groter dan O(bol). Het hangt niet af van de grootte van de straal r van de bol.

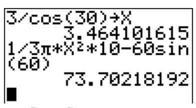
D8a $I(\text{cilinder}) = G \cdot h = \pi r^2 h = \pi \cdot 5^2 \cdot 10 = \pi \cdot 25 \cdot 10 = 250\pi \approx 785$ (cm³).
D8b $I(\text{bol}) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{4}{3}\pi \cdot 125 = \frac{500}{3}\pi \approx 524$ (cm³).

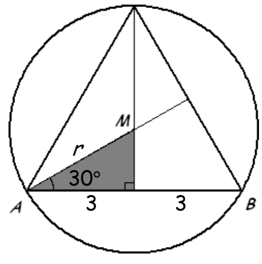


D9a $I(\text{piramide}) = \frac{1}{3} G \cdot h = \frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin(60°) \cdot 10 = 6 \cdot \sin(60°) \cdot 10 = 60 \cdot \sin(60°) \approx 51{,}96$.
D9b $I(\text{afgeknotte piramide}) = \frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin(60°) \cdot 10 - \frac{1}{3} \cdot \frac{1}{2} \cdot 3{,}6 \cdot 3{,}6 \cdot \sin(60°) \cdot 6 \approx 40{,}74$.
(de maten in de toppiramide die eraf gehaald is, zijn $\frac{6}{10}$ deel van de maten in gegeven priramide)



D9c Zie het bovenaanzicht (dit is het grondvlak) hiernaast. (T ligt precies boven M)
In de grijs gemarkeerde driehoek is: $\cos(30°) = \frac{3}{r} \Rightarrow r = \frac{3}{\cos(30°)}$.
$I(\text{kegel}) = \frac{1}{3} G \cdot h = \frac{1}{3}\pi r^2 \cdot h = \frac{1}{3}\pi \cdot (\frac{3}{\cos(30°)})^2 \cdot 10$.
$I(\text{kegel buiten de piramide}) = \frac{1}{3}\pi \cdot (\frac{3}{\cos(30°)})^2 \cdot 10 - 60 \cdot \sin(60°)$ (zie D9a) $\approx 73{,}70$.



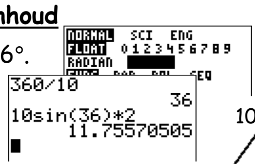


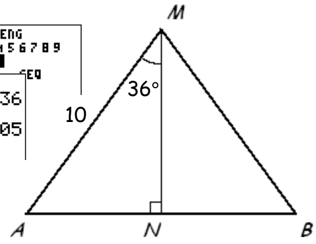
Gemengde opgaven 2. Oppervlakte en inhoud

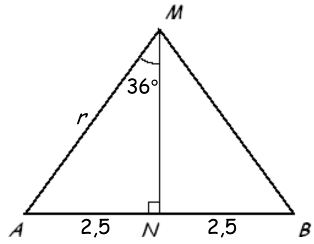
G13a $\angle AMB = \frac{360°}{5} = 72° \Rightarrow \angle AMN = \frac{\angle AMB}{5} = 36°.$

$\sin(36°) = \frac{AN}{r} = \frac{AN}{10} \Rightarrow AN = 10\sin(36°).$

Dus $a = AB = 2AN = 20\sin(36°) \approx 11,8.$



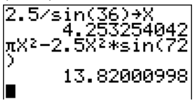




G13b $O(ABCDE) = 5 \cdot O(\Delta ABM) = 5 \cdot \frac{1}{2} \cdot r \cdot r \cdot \sin(72°).$

$\sin(36°) = \frac{AN}{r} = \frac{2,5}{r} \Rightarrow r = \frac{2,5}{\sin(36°)}.$

$O(\text{blauw}) = \pi r^2 - 2,5r^2 \cdot \sin(72°) \approx 13,8.$

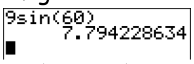


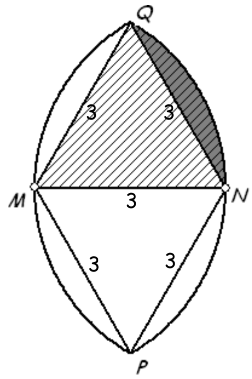
G14a Noem M het middelpunt van c_1 en N het middelpunt van c_2. (dan eenvoudiger te noteren)

Er geldt $MN = MQ = MP = MQ = NQ = r = 3.$

Het parallellogram bestaat uit 2 gelijkzijdige driehoeken. Zie de figuur hiernaast.

$O(MPNQ) = 2 \cdot O(\Delta MNQ) = 2 \cdot \frac{1}{2} \cdot 3 \cdot 3 \cdot \sin(60°) = 9\sin(60°) \approx 7,79.$

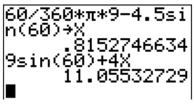




G14b De gevraagde oppervlakte is de oppervlakte van het parallellogram (zie G14a) en daarnaast nog 4 keer de oppervlakte van het grijs gemarkeerde segment.

$O(\text{segment}) = O(\text{setor } MNQ) - O(\Delta MNQ) = \frac{60}{360} \cdot \pi \cdot 3^2 - \frac{1}{2} \cdot 3 \cdot 3 \cdot \sin(60°).$

$O(\text{overlapping in figuur } G.8) = O(MPNQ) + 4 \cdot O(\text{segment}) \approx 11,06.$

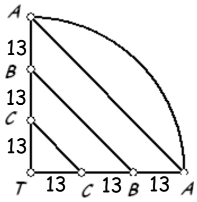


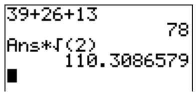
G15 De lengte van het langste lint is: $AA = \sqrt{39^2 + 39^2} = \sqrt{39^2 \cdot 2} = 39\sqrt{2}$ (cm).

De lengte van het middelste lint is: $BB = \sqrt{26^2 + 26^2} = \sqrt{26^2 \cdot 2} = 26\sqrt{2}$ (cm).

De lengte van het kortste lint is: $CC = \sqrt{13^2 + 13^2} = \sqrt{13^2 \cdot 2} = 13\sqrt{2}$ (cm).

De totale lengte van de drie linten is: $39\sqrt{2} + 26\sqrt{2} + 13\sqrt{2} \approx 110$ (cm).

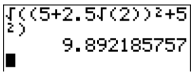


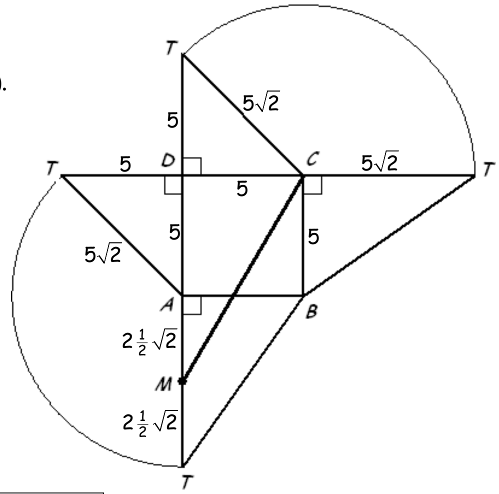


G16a Zie de uitslag hiernaast. (er is langs de opstaande ribben opengeknipt)

G16b In ΔADT: $AT = \sqrt{5^2 + 5^2} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$ (cm) $\Rightarrow AM = \frac{1}{2}AT = 2\frac{1}{2}\sqrt{2}$ (cm).

In ΔCDM: $CM = \sqrt{(5 + 2\frac{1}{2}\sqrt{2})^2 + 5^2} \approx 9,9$ (cm).





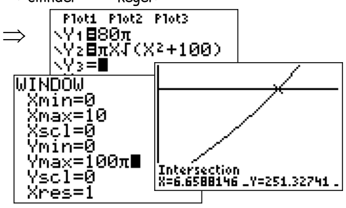
G17 $O(\text{cilindermantel}) = 2\pi rh = 2\pi \cdot 4 \cdot 10 = 80\pi.$ ($r_{\text{cilinder}} \neq r_{\text{kegel}}$)

Bij de kegel is $R = \sqrt{r^2 + h^2} = \sqrt{r^2 + 100} \Rightarrow$

$O(\text{kegelmantel}) = \pi rR = \pi \cdot r \cdot \sqrt{r^2 + 100}.$

Er geldt dus: $80\pi = \pi \cdot r \cdot \sqrt{r^2 + 100}.$

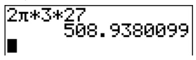
De optie intersect geeft dan $r \approx 6,66.$



G18 Het pijpje is te maken van een cilinder met $h = 12 + 9 + 2 \cdot 3 = 27$ (cm).

(denk de versteknaad los en stukjes pijp in elkaars verlengde tegen elkaar aan)

$O(\text{pijpje}) = O(\text{cilindermantel}) = 2\pi rh = 2\pi \cdot 3 \cdot 27 = 162\pi \approx 509$ (cm²).



G 19 De emmer is een afgeknotte kegel. De hoogte van de hele kegel is $h = 3 \cdot 35 = 105$ (cm).

(de diameter aan de bovenkant van de emmer is 33 cm; 35 cm lager is de diameter nog 22 cm, dus 11 cm minder;
weer 35 cm lager is de diameter nog 11 cm en nog één keer 35 cm lager is de diameter dan 0 cm ⇒ dit is de top van de kegel)

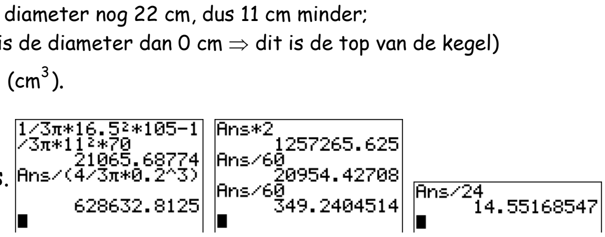
$I(\text{emmer}) = I(\text{KEGEL}) - I(\text{kegel}) = \frac{1}{3}\pi \cdot (\frac{33}{2})^2 \cdot 105 - \frac{1}{3}\pi \cdot (\frac{22}{2})^2 \cdot 70$ (cm³).

$I(\text{druppel}) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 0,2^3$ (cm³).

In de emmer passen $\dfrac{\frac{1}{3}\pi \cdot 33^2 \cdot 105 - \frac{1}{3}\pi \cdot 22^2 \cdot 70}{\frac{4}{3}\pi \cdot 0,2^3} \approx 629\,000$ druppels.

Het duurt dus ongeveer $1\,258\,000$ seconden.

Dat is Ans : 60 (minuten) : 60 (uur) : 24 (dagen) $\approx 14,6$ dagen. Dus ruim 2 weken.



G20 $I(\text{onderste helft}) = I(\text{KEGEL}) - I(\text{kegel}) = \frac{1}{3}\pi \cdot 2^2 \cdot 6 - \frac{1}{3}\pi \cdot 1^2 \cdot 3 = 8\pi - \pi = 7\pi.$

Dus $I(\text{diabolovormige lichaam}) = 2 \cdot 7\pi = 14\pi.$

G21a $I(\text{piramide}) = \frac{1}{3}Gh = \frac{1}{3}(4 \cdot 6 + \frac{1}{2} \cdot 6 \cdot 2) \cdot 10 = \frac{1}{3}(24+6) \cdot 10 = \frac{1}{3} \cdot 30 \cdot 10 = 10 \cdot 10 = 100.$

G21b Op hoogte 3 (7 onder de top) zijn de lengtematen in het hor. vlak $\frac{7}{10}$ deel van de lengtematen in vijfhoek $ABCDE$.
Op hoogte 5 (5 onder de top) zijn de lengtematen $\frac{5}{10}$ deel (de helft) van die in vijfhoek $ABCDE$.
$I(\text{vraag 21b}) = \frac{1}{3}(\frac{7}{10} \cdot 4 \cdot \frac{7}{10} \cdot 6 + \frac{1}{2} \cdot \frac{7}{10} \cdot 6 \cdot \frac{7}{10} \cdot 2) \cdot \frac{7}{10} \cdot 10 - \frac{1}{3}(\frac{1}{2} \cdot 4 \cdot \frac{1}{2} \cdot 6 + \frac{1}{2} \cdot \frac{1}{2} \cdot 6 \cdot \frac{1}{2} \cdot 2) \cdot \frac{1}{2} \cdot 10 = 21{,}8.$

```
1/3*(2.8*4.2+1/2
*4.2*1.4)*7-1/3*
(2*3+1/2*3*1)*5
            21.8
0.7^3*100-0.5^3*
100
            21.8
```

G22a $\sin\alpha = \frac{r}{R} \Rightarrow R = \frac{r}{\sin\alpha}$ invullen in $O(\text{kegelmantel}) = \pi r R$ geeft $O(\text{kegelmantel}) = \pi r \cdot \frac{r}{\sin\alpha} = \frac{\pi r^2}{\sin\alpha}$.
$O(\text{kegelmantel}) = O(\text{cirkelsector}) = \frac{\beta}{360°} \cdot \pi R^2 \Rightarrow \frac{\beta}{360°} \cdot \pi R^2 = \pi r R \Rightarrow \beta = 360° \cdot \frac{\pi r R}{\pi R^2} = 360° \cdot \frac{r}{R} = 360° \cdot \sin\alpha.$

G22b $O(\text{kegelmantel}) = \pi r R$ maar ook $O(\text{kegelmantel}) = \frac{\beta}{360°} \cdot \pi R^2$.
Dus $\pi r R = \frac{\beta}{360°} \cdot \pi R^2 \Rightarrow \pi r \cdot 10 = \frac{135°}{360°} \cdot \pi \cdot 10^2 \Rightarrow r = \frac{135}{360} \cdot 10 = \frac{15}{4}$.
Dus $O(\text{grondcirkel}) = \pi r^2 = \pi \cdot \left(\frac{15}{4}\right)^2 = \frac{225}{16}\pi \approx 44{,}2.$

```
135/360*10▸Frac
            15/4
Ans²▸Frac
          225/16
Ans*π
     44.17864669
```

G22c $\left.\begin{array}{l} O(\text{kegel}) = O(\text{grondcirkel}) + O(\text{kegelmantel}) \\ O(\text{kegel}) = 4 \cdot O(\text{grondcirkel})\ (\text{gegeven}) \end{array}\right\} \Rightarrow 4 \cdot O(\text{grondcirkel}) = O(\text{grondcirkel}) + O(\text{kegelmantel}).$
Dus $3 \cdot O(\text{grondcirkel}) = O(\text{kegelmantel}).$
Dit geeft $3 \cdot \pi r^2 = \frac{\pi r^2}{\sin\alpha} \Rightarrow 3 = \frac{1}{\sin\alpha} \Rightarrow \sin\alpha = \frac{1}{3} \Rightarrow \alpha = \sin^{-1}(\frac{1}{3}) \Rightarrow$ de tophoek is $2\alpha = 2\sin^{-1}(\frac{1}{3}) \approx 39°.$

```
sin⁻¹(1/3)
     19.47122063
Ans*2
     38.94244127
```

TI-84 4. Goniometrie en geheugen

1a $10\sin(15°) \approx 2{,}59.$

1b $\frac{20}{\cos(15°)} \approx 20{,}71.$

```
NORMAL  SCI  ENG
FLOAT 0123456789
RADIAN
10sin(15)
     2.588190451
20/cos(15)
     20.70552361
```

1c $\frac{1}{2} \cdot 7^2 \cdot \sin(72°) \approx 23{,}30.$

1d $\frac{12}{\tan(15°) + \tan(35°)} \approx 12{,}39.$

```
1/2*7²*sin(72)
     23.30088465
12/(tan(15)+tan(
35))
     12.39468737
```

2a $\angle A = \sin^{-1}\left(\frac{3}{7}\right) \approx 25°.$

2b $\angle A = \cos^{-1}\left(\frac{\sqrt{53}}{8}\right) \approx 24°.$

```
sin⁻¹(3/7)
     25.37693353
cos⁻¹(√(53)/8)
     24.49275017
```

2c $\angle A = \tan^{-1}\left(\frac{1{,}6}{\sqrt{13}}\right) \approx 24°.$

2d $\angle A = \sin^{-1}\left(\frac{5+\sqrt{6}}{8+\sqrt{15}}\right) \approx 39°.$

```
tan⁻¹(1.6/√(13))
     23.92974098
sin⁻¹((5+√(6))/(8
+√(15)))
      38.8609136
```

3a $\angle A = \frac{12}{\sqrt{13}}$ en $PQ = \frac{6}{\tan\angle A} \Rightarrow KL = 12 \cdot (5 + 3PQ) \approx 908{,}29.$

```
100/7
     14.28571429
6/tan(Ans)
     23.56348289
12*(5+3Ans)
     908.2853839
```

3b $\tan\angle E = \frac{12}{\sqrt{13}}$ en $CD = \frac{5{,}17}{\sin\angle E} \Rightarrow AB = \frac{80}{6 + CD^2} \approx 2{,}28.$

```
tan⁻¹(12/√(13))
     73.27644199
5.17/sin(Ans)
     5.398326194
80/(6+Ans²)
     2.276483102
```

$A = 2{,}123456;\quad B = \sqrt{3}$ en $C = \frac{4\pi}{10{,}3498}.$

```
2.123456→A
        2.123456
√(3)→B
     1.732050808
4π/10.3498→C
      1.21416555
```

4a $3A - B \approx 4{,}638.$

4b $C^2 + AB \approx 5{,}152.$

```
3A-B
     4.638317192
C²+AB
     5.152131664
```

4c $B^2 - 4AC \approx -7{,}313.$

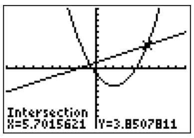
4d $\frac{C^2 - AB}{A+B} \approx -0{,}572.$

```
B²-4AC
    -7.312908493
(C²-AB)/(A+B)
    -.5715813266
```

5a Voer de formule in op de GR. In het derde scherm (hieronder) zie je de coördinaten van het rechter snijpunt.

```
Plot1 Plot2 Plot3
\Y1■0.5X²-2X-1
\Y2■0.5X+1
\Y3=
\Y4=
```

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```



5b De coördinaten van het laatst gevonden punt in het grafiekenscherm kunnen in het basisscherm met X en Y worden opgeroepen.

```
X
     5.701562119
Y
     3.850781059
```